Available online at www.sciencedirect.com

ScienceDirect

Physics Procedia 70 (2015) 326 – 329

Physics Procedia

2015 International Congress on Ultrasonics, 2015 ICU Metz

Interaction of a Guided Wave with a Crack in an Embedded Multilayered Anisotropic Plate: Global Matrix with Laplace Transform Formalism

Pierric Mora[a,*], Eric Ducasse[a,b], Marc Deschamps[a,c]

[a] *University of Bordeaux, I2M-APy, UMR 5295, F-33400 Talence, France*
[b] *Arts et Metiers ParisTech, I2M-APy, UMR 5295, F-33400 Talence, France*
[c] *CNRS, I2M-APy, UMR 5295, F-33400 Talence, France*

Abstract

We solve the problem of the interaction of a transient guided elastic wave by a planar crack with an indirect Boundary Element approach in the Laplace domain ($t \to s$). The originality of this work is to use the numerical Green function of the layered plate rather than the analytical Green function of each layer. As a consequence, the BEM matrices are small. To obtain the Green function in the (x,z,s) domain we first solve the equations in the Fourier transform (k,z,s) domain with a Global Matrix approach, and then perform a numerical inverse FFT. Comparisons with finite element show excellent agreement. This approach is fast and low memory consuming for planar defects in arbitrary layered media, and can be extended to arbitrary shapes and boundary conditions for a higher computational cost. It is valid in 3D, however only the 2D case is considered in this work.

Peer-review under responsibility of the Scientific Committee of ICU 2015

Keywords: embedded plate, anisotropic, Boundary Element Method, horizontal crack, Green function, Global Matrix, Laplace, transient wave

1. Principle

Hybrid methods based on normal modes theory coupled to Finite Element are very powerful tools for solving the problem of the interaction of an incident wave with a defect located in a plate. However, to the day, then still cannot be used in every type of waveguide. Indeed, difficulties arise when dealing with 3D general anisotropy, and with open waveguides. A solution could be to build a hybrid method based on partial waves (Lowe (1995)), because such numerical approach does not require to find the normal modes. Therefore, it is much easier to deal with arbitrarily layered media. Surprisingly, there is little literature on it. Wang and Gross (2001) proposed a method inspired from the analytical expression of the integral singular equation, which is valid only if the defect is a planar crack. Vacossin et al. (2009) proposed an implementation of the Kirchhoff approximation for the case of planar defects, and Bedrici et al. (2009) designed an iterative process to get rid of the approximation and allow more general shapes. However, the convergence of the iterative process breaks if the impedance mismatch between the plate and the surrounding infinite

* Corresponding author. Tel.: +33-540-00-87-87
E-mail address: pierric.mora@u-bordeaux.fr

Peer-review under responsibility of the Scientific Committee of ICU 2015
doi:10.1016/j.phpro.2015.08.217

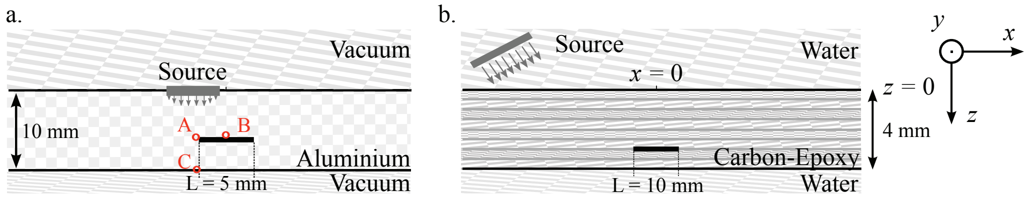

Fig. 1. a. Aluminium cracked plate in vacuum; b. Carbon-Epoxy plate with an adhesive disbond immersed in water

medium is too high. In this work we propose to compute the semi analytical Green function of the layered medium in the Laplace transform domain to feed a Boundary Element Method. Therefore, there is no issue of iterations, physical approximation, or limitation to canonical geometries. To the best of our knowledge, this is original.

1.1. Equation to solve

Let us first consider that the layered medium is free of defect. A given source generates an elastodynamic field which will be refered as incident field. Then, consider the presence of a defect. According to the Huygens-Fresnel principle, the defect acts as a secondary source distribution which satisfies the following equation:

$$\int_0^t \int_{-\frac{L}{2}}^{\frac{L}{2}} (\mathbf{e}_z \diamond \nabla)\left(\mathbf{g}(x-\tilde{x}, z_c, t-\tilde{t}):\mathbf{f}_m(\tilde{x},\tilde{t}) + \partial_z \mathbf{g}(x-\tilde{x}, z_c, t-\tilde{t}):\mathbf{f}_b(\tilde{x},\tilde{t})\right) d\tilde{x}\, d\tilde{t} = -\boldsymbol{\sigma}_z^{(i)}(x, z_c, t),\ \ \forall x \in \left]-\frac{L}{2}, \frac{L}{2}\right[, \quad (1)$$

where z_c is the depth of the crack, $\mathbf{e}_z$ is the unitary vector along the z axis, $\diamond$ is a bilinear operator such that $(\mathbf{a} \diamond \mathbf{b})_{il} = c_{ijkl} a_j b_k$ (Ducasse and Deschamps (2012)), $\mathbf{g}$ is the (known) causal Green tensor of the embedded plate without the crack, $\boldsymbol{\sigma}_z^{(i)}$ is the (known) incident stress field normal to the crack and $\mathbf{f}_m$ and $\mathbf{f}_b$ are the (unknown) density fields of secondary monopolar and bipolar sources (Sutradhar et al. (2008)). For this specific case of a Neumann condition, $\mathbf{f}_m = \mathbf{0}$. We transform (1) into the Laplace domain ($t \to s$) to replace the time convolution by an equation to solve for each s. Upper case letters in the Laplace domain replace the lower case letters in the time domain. For convenience, the dependence of the functions along the s variable is omitted.

1.2. Mesh of the crack

We split the unknown density $\mathbf{F}_b(x)$ into $3N$ small elements of unknown scalar amplitude $A_{n,x}$, $A_{n,y}$ and $A_{n,z}$, identical shape $\psi(x)$ and constant spacing. Usually one uses a simple piecewise polynomial shape function such as a triangular door. In this work, the contribution of each element is to be calculated by a spatial Fourier transform. For convenience we choose the shape function to be a Gaussian door, in order to have a faster decreasing spectrum.

$$\begin{aligned} &\mathbf{F}_b(x) \approx \textstyle\sum_{n=0}^{N-1} \psi(x - \frac{nL}{N}) \left(A_{n,x} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T + A_{n,y} \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T + A_{n,z} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T \right). \\ &\psi = exp\left(-\frac{\tilde{x}^2}{2\sigma^2}\right) * \Pi\left(\frac{L}{N}\left(\tilde{x} - \frac{1}{2}\right)\right) \end{aligned} \quad (2)$$

1.3. Green tensor of the layered medium

We use a Global Matrix approach to solve the wave equation in the Fourier-Laplace transform space (k, z, s). Indeed, this is the fastest way to get the response of a transient source in the near field domain. Therefore, we never get the Green tensor in the (x, z, s) space, because this would imply to sample the space with an infinitely thin step. We rather get directly the contribution of any element in the (k, z, s) space, which has a limited spectrum range controlled by σ, and then perform a numerical inverse Fourier transform to the (x, z, s) space. We shall therefore define:

$$\begin{aligned} \boldsymbol{\Sigma}_z^{(0,x)} &= (\mathbf{e}_z \diamond \nabla)\, \partial_z \mathbf{G}(\bullet, z_c) : \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}^T * \psi \\ \boldsymbol{\Sigma}_z^{(0,y)} &= (\mathbf{e}_z \diamond \nabla)\, \partial_z \mathbf{G}(\bullet, z_c) : \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T * \psi \\ \boldsymbol{\Sigma}_z^{(0,z)} &= (\mathbf{e}_z \diamond \nabla)\, \partial_z \mathbf{G}(\bullet, z_c) : \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T * \psi. \end{aligned} \quad (3)$$

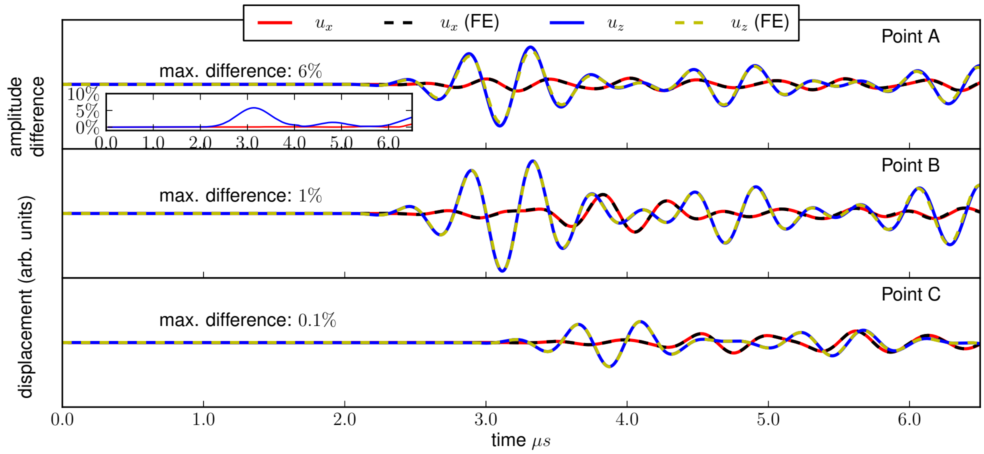

Fig. 2. Aluminium plate: comparison with a Finite Element (FE) computation

Notice that only 4 contributions are needed to assemble the BEM matrix: the direct field $\mathbf{\Sigma}_z^{(i)}$ and the contributions of the first three elements $\mathbf{\Sigma}_z^{(0,x)}$, $\mathbf{\Sigma}_z^{(0,y)}$ and $\mathbf{\Sigma}_z^{(0,z)}$. According to the invariance along the x direction, all other element contributions $\mathbf{\Sigma}_z^{(n,x)}$, $\mathbf{\Sigma}_z^{(n,y)}$ and $\mathbf{\Sigma}_z^{(n,z)}$ are deduced from the former ones.

1.4. Resolution of the Boundary Element system

Following a naive collocation method, the boundary integral equation is sampled at N points chosen to be the centers of the elements. This leads to a linear system of $3N$ equations and $3N$ unknowns (4). This system is solved for each s, and the field can be reconstructed by performing a regular computation. Then, an inverse Laplace transform is performed numerically.

$$\sum_{n=0}^{N-1} A_{n,x}\,\mathbf{\Sigma}_z^{(0,x)}((m-n)\,d, z_c) + A_{n,y}\,\mathbf{\Sigma}_z^{(0,y)}((m-n)\,d, z_c) + A_{n,z}\,\mathbf{\Sigma}_z^{(0,z)}((m-n)\,d, z_c) = -\mathbf{\Sigma}_z^{(i)}(m\,d, z_c), \quad m = 0\,..\,N-1 \tag{4}$$

2. Numerical examples

Two examples are considered. The first one is presented for validation purpose and consists of a cracked aluminium plate in vacuum (see fig. 1 a.), insonified by a surface transducer. The second example is related to a common NDT situation: the modal inspection of a carbon-epoxy plate having an adhesive disbond (see fig. 1 b.).

2.1. Aluminium plate in vacuum

Elastic constants of the aluminium: $\rho = 2.870\,g\,mm^{-3}$, $\lambda = 58.0\,GPa$, $\mu = 27.0\,GPa$. A 5 cycles tone burst ($f = 2.25\,MHz$) is sent by a surface transducer. The crack is meshed with $N = 200$ elements and $\sigma = 0.025\,mm$. Computation time: $8\,s$. A comparison is made with a Finite Element computation (see fig. 2). As expected, the differences between the BEM and FE computations are maximal in the vicinity of the crack, especially near the edges. However, they never exceed a few percents. Farther of the crack the differences are smaller by one order of magnitude.

2.2. Carbon-Epoxy plate in water

The plate is made of 8 layers stacked together. Each layer is $0.5\,mm$ thick, has the physical constants described below and is rotated in the interfaces plane according to the sequence: $[0°, 135°, 90°, 45°, 0°, 135°, 90°, 45°]$. A 5

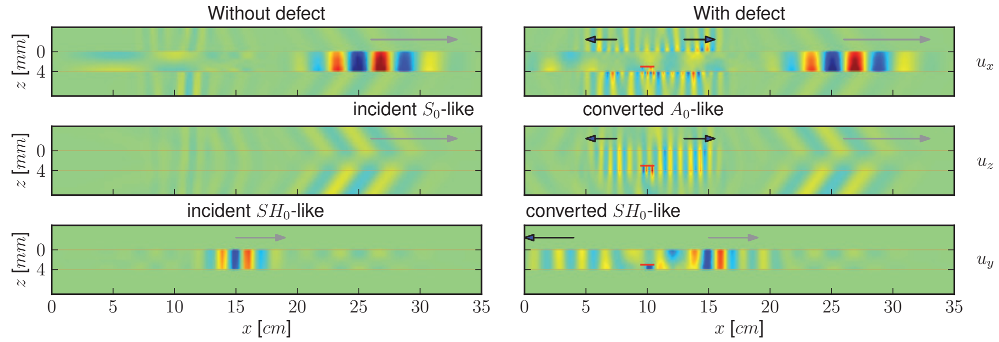

Fig. 3. Carbon-Epoxy plate with an adhesive disbond immersed in water: displacement field after the interaction

cycles tone burst ($f = 150.0\,kHz$) is sent by a transducer located in the water, which is tilted to select principally a S_0-like and a SH_0-like modes in the plate. As can be seen in fig. 3, these modes interact with the defect by producing mainly a A_0-like and a SH_0-like modes. The converted A_0-like radiates its energy to the fluid well enough to be detectable. The crack is meshed with $N = 40$ elements and $\sigma = 0.25\,mm$. Computation time: $1\,min$. Physical constants used: Water: $\rho = 1.0\,g\,mm^{-3}$, $c = 1.5\,mm\,\mu s^{-1}$. Carbon-Epoxy: $\rho = 1.560\,g\,mm^{-3}$, $c_{11} = 86.60 + i\,7.50$, $c_{22} = 13.50 + i\,0.60$, $c_{33} = 14.00 + i\,0.28$, $c_{44} = 2.72 + i\,0.10$, $c_{55} = 4.06 + i\,0.12$, $c_{66} = 4.70 + i\,0.28$, $c_{12} = 9.00 + i\,0.30$, $c_{13} = 6.40 + i\,0.60$, $c_{23} = 6.80 + i\,0.25\,GPa$. Hysteretic model is used for viscous part.

3. Conclusions

This approach is of the most efficient ones that can be used to model the interaction of planar defects in layered waveguides with a transient incoming wave. The naive implementation described in this work can be significantly improved using well-known techniques, such as a Galerkin method to build the BEM matrix and a refinement of the mesh near the edges. Furthermore, arbitrary linear boundary conditions can be considered. An extension is possible for arbitrary shapes and 3D. This approach could also be a way of building numerical transparent boundary conditions to be used by Finite Element. Therefore, the very general case of an arbitrary defect in an arbitrarily anisotropic, viscoelastic, embedded plate could be solved in the Laplace transform domain by a hybrid method.

Acknowledgements

This work is funded by Region Aquitaine and CEA-LIST.

References

Lowe, M. J. S., 1995. Matrix Techniques for Modeling Ultrasonic Waves in Multilayered Media. IEEE Trans. Ultrasonics Ferroel. Freq. Contr. 42, 525–542.

Wang, Y. S., Gross, D., 2001. Transfer matrix method of wave propagation in a layered medium with multiple interface cracks: Antiplane case. J. Appl. Mech. 68, 499–503.

Vacossin, B., Potel, C., Gatignol, P., de Belleval, J. F., 2009. Interaction of a monochromatic ultrasonic beam with a finite length defect at the interface between two anisotropic layers - Kirchhoff approximation and Fourier representation. IEEE Trans. Ultrasonics Ferroel. Freq. Contr. 56, 2251–2267.

Bedrici, N., Gatignol, P., Potel, C., 2009. An iterative method for the interaction between a bounded beam and an interface defect in solids, under Kirchhoff approximation. Acta Acustica united with Acustica 95, 189–202.

Ducasse, E., Deschamps, M., 2012. A nonstandard wave decomposition to ensure the convergence of Debye series for modeling wave propagation in an immersed anisotropic elastic plate. Wave Motion, 49, 745–764.

Sutradhar, A., Paulino, G. H., Gray, L. J., 2008. Symmetric Galerkin Boundary Element Method. Springer-Verlag Berlin Heidelberg